International Journal of Scientific & Engineering Research, Volume 3, Issue 9, September-2012
ISSN 2229-5518

Order of Stress in a Homogeneous Field

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Abstract— In a structure the presence of a geometric discontinuity of order or material reduces significantly its lifetime. Indeed, it is at these areas that the stress field is strongly disturbed and can reach very high critical levels resulting a damage process. The analysis of these unique areas can not only achieve an optimal design of the geometry of structures but also can choose materials for various assemblies. This analysis necessarily involves determining the parameters of the asymptotic uniqueness constraints. These parameters are the stress intensity factor, the tensor and standardized the order of singularity. This last parameter that indicates the severity of the singularity. It is directly connected to the geometric aspect, and incompatible materials present in the structure. This study focuses on the identification and analysis of this important parameter in the reliability of structures made of homogeneous materials.

Index Terms— Geometric Discontinuity, Stress Intensity, Severity, Singularity, Optimal Design.

1 Introduction

In structure, the singularities of stresses can be developed from either a lack of geometric regularity or a discontinuity between the materials involved. It is at the level of these unique areas that the risk of damage is paramount. The analysis of these singularities constraints requires the determination of asymptotic quantities [1,2]. We recall that these quantities are: stress intensity factor related to the nature of the materials and the type of load applied, tensor which is a standard function space to illustrate the stress distribution and finally, the order of the singularity parameter reflecting the severity of the discontinuity involved. In this study, we will focus particularly on identifying and analyzing the order of singularity for structures made of homogeneous materials. We started from the classical theory of elasticity to make this determination numerically. And at the same time of this theoretical approach, we developed an iterative calculation based on the finite element method. Both methods have been applied to various cases of singularities form of cuts made on a three-point bending test and the results have been analyzed and compared with literature.

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2 Theoretical approach

The analysis of stress field near a singularity (edge, hole ...) becomes possible by asymptotic methods. The first asymptotic analyses have been developed by Williams [3] then by Bogy [1] and others later. These authors have established a formulation showing the singularity near edge. When the edge distance r is small compared to other geometric characteristic lengths, the stress field has the form kr-λ, λ one of asymptotic quantities, it is called the exponent that expresses the severity of singularity. Its value is in the range 0 <λ <1, according to the angles of the opening. The intensity factor constraint k can not be determined by the asymptotic analysis, but by analyzing the digital field for material geometry and specific loading. Through the theory of planar linear elasticity, we will analyze the order of singularity depending on the notch in the area described in Fig 1

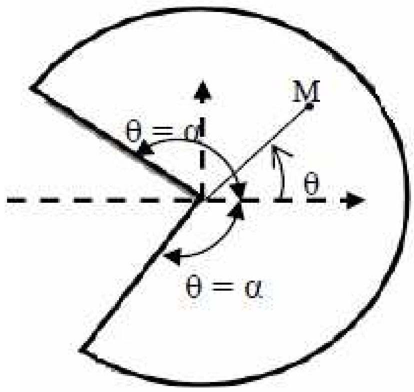

Fig.1 Homogeneous field with a singularity opening 2π-2α

In this area, the displacement field is singular. In polar coordinates, the components of this field can be expressed as follows [4,5]:

$$u_r(r,\theta) = r^{\lambda} f(\theta),\ u_\theta(r,\theta) = r^{\lambda} g(\theta). \quad (1)$$

We recall that these components satisfy the following equations of equilibrium:

$$\begin{cases} (\lambda+2\mu)\dfrac{\partial}{\partial r}\left(\dfrac{\partial u_r}{\partial r}+\dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta}+\dfrac{u_r}{r}\right)-\mu\dfrac{1}{r}\dfrac{\partial}{\partial \theta}\left(\dfrac{\partial u_\theta}{\partial r}-\dfrac{1}{r}\dfrac{\partial u_r}{\partial \theta}+\dfrac{u_\theta}{r}\right)=0 \\ (\lambda+2\mu)\dfrac{1}{r}\dfrac{\partial}{\partial \theta}\left(\dfrac{\partial u_r}{\partial r}+\dfrac{1}{r}\dfrac{\partial u_\theta}{\partial \theta}+\dfrac{u_r}{r}\right)+\mu\dfrac{\partial}{\partial r}\left(\dfrac{\partial u_\theta}{\partial r}-\dfrac{1}{r}\dfrac{\partial u_r}{\partial r}+\dfrac{u_\theta}{r}\right)=0 \end{cases}$$

(2)

The search for nontrivial solutions of this equation system led to the establishment of called transcendental equations. These are the equations that will enable to highlight the dependence of the order of singularity depending on the geometry studied through its opening angle. The main dependencies expressed by the characteristic equations have been established for different types of boundary conditions [5]. Given the analytical resolution of these very time-consuming characteristic equations, we considered a numerical approach. In this approach we have developed appropriate programs on MATLAB. So for each fixed α, we seek solution of the equation λ considered.

In this study, we investigate the case of the lips of notch free of stress whose characteristic equation is given by:

$$\sin 2\lambda\alpha = \pm\lambda \sin 2\alpha \quad (3)$$

The dependence between the notch and the exponent of the singularity, expressed by this equation is illustrated by the curves of Fig 2.

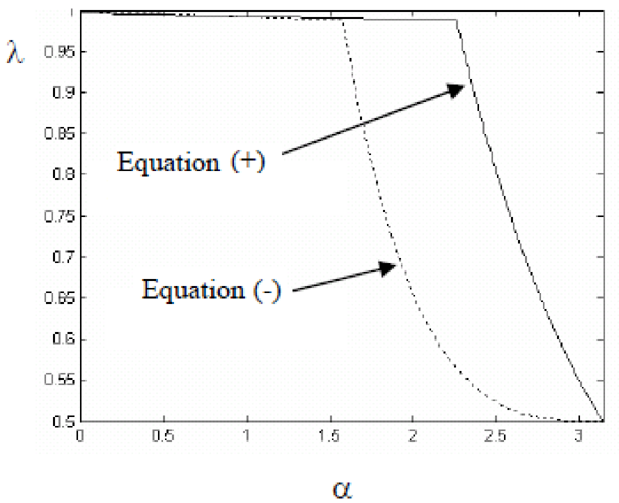

Fig. 2 Development of the exponent λ in terms of angle α for a singularity with lips of notch free from stress

Through the evolution of these curves we can see that for the characteristic equation with the sign (-), the mechanical behavior is regular as α remains less than 1.5 rad. Beyond this value, the singular behavior begins to develop and is evolving rapidly to a critical case of the crack for which the exponent λ takes the value λ = 0.5. In contrast to the equation with the sign (+), the singular behavior appears only to an angle α = 2.4 rad and the progression to the limiting value of λ = 0.5 is faster compared with the case of the sign (-).

3 Numerical approach

For the area shown in Fig 1, we recall that the components of the stress field at point M with polar coordinates (ρ, θ) near the singular point, expressed from the following relationship [6,7]:

$$\sigma_{ij}(\rho,\theta) = kf_{ij}(\theta)\rho^{-\lambda} \quad (4)$$

With λ is taken between 0 <λ <1 which implies that the stress increases as the radius ρ decreases, the exponent depends on the geometry of field around the singular point. This parameter characterizes the severity of the singularity. k is the stress intensity factor, depending mainly on the type and intensity of loading, and elastic properties of the material near the singularity. fij (θ) is the normalized tensor reflecting the stress variations with polar angle θ.
To determine these asymptotic quantities, so we will consider a numerical approach based on an iterative finite element method, [8]. The principle of this method can be summarized through the following steps:

- building an area A0 around the singular point O (Fig 3), with invariants border Γ0 and mesh inside an homothety of center O and ratio p (0 <p <1);
- Reducing the area A0 in areas Ai with contours Γi increasingly smal;
- Calculating the displacements, strains and stresses in each mesh element.

$$u_i(M_0) = \frac{1}{p} u_{i-1}(M_1) \quad (5)$$

With M0 and M1 are two points belonging respectively to the border Γ0 and Γ1 and using the finite element method we obtained :

$$u_i = \frac{1}{p^i} u_0 \quad (6)$$

ISSN 2229-5518

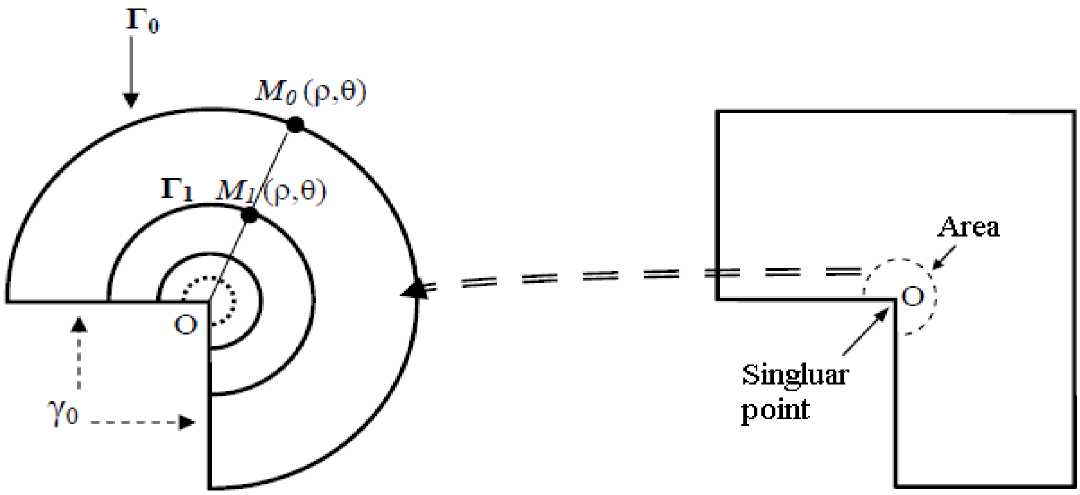

Fig. 3 Schematic of a structure with a singular point
a) Area A of the structure with O singular point.
b) Local Area A0 included in the domain A, and having Γ0 and γ0 as the boundary)

We assume that the area A0 defined before is reduced in n subdomain obtained by homothetic transformation of ratio p. The iterative finite elements successively determine the displacements and stresses for these reduced areas. Expressing the equation (1) for two successive iterations n and n-1, we can calculate the exponent λ by the following expression:

$$\lambda = \frac{1}{\ln(p)} \ln\left[\frac{\sigma_n(\rho_0,\theta)}{\sigma_{n-1}(\rho_0,\theta)}\right] \quad (7)$$

4 Application

For the approach outlined above, we developed a C++ program allowing each iteration to use the results in displacements and stresses obtained by SAMCEF or ANSYS. We considered the study of a specimen with a notch opening (ω) and subjected to a three-point bending Fig 4. The material of this specimen is a steel whose modulus E = 2.3 GPa and Poisson's ratio v = 0.36.

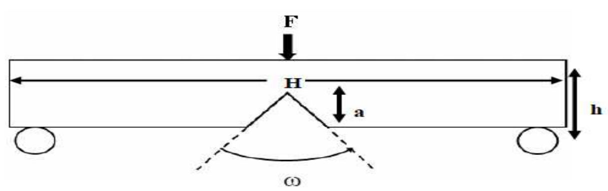

Fig. 4 Specimen of opening ω subject to 3 points bending
(H = 50 mm, h = 6 mm and 1.5 mm)

For this specimen, we performed for each case of opening ω a first modeling considering the overall structure of Fig 5a. As a result we developed the iterative calculations in a local domain around the singularity Fig 5b.

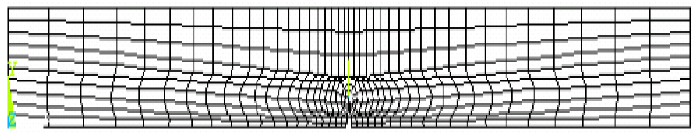

Fig. 5(a) Mesh of the overall structure for ω = 15 °

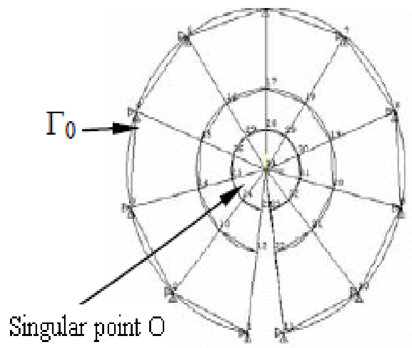

Fig. 5(b) Mesh of the local area around the singular point O

In Table 1 we have summarized the results we obtained for the exponent λ by the numerical approach and the theoretical approach. In the same table we also reproduce the literature values [9]

TABLE 1
Comparison of theoretical values, Numerical and literature of the singularity exponent λ

	the singularity exponent λ		
ω	Theoretical approach	Numerical approach	D. Leguillon [9]
30°	0.512	0.539	0.502
45°	0.517	0.551	0.506
60°	0.528	0.548	0.513
90°	0.558	0.562	0.545
120°	0.613	0.652	0.616
150°	0.718	0.752	0.752

4 Summary

Through this study, we developed two approaches, one theoretical and one numerical to determine the exponent of the singularity of a homogeneous field with a notch. The results we obtained for this exponent by the two mentioned approaches are in good agreement. And validation of these methods has been supported by the results obtained by [9]. The determination of the asymptotic parameter allows a better reporting of the severity of the singularity that may have adverse effects on the mechanical behavior of the structure. Therefore the design of such structures, corrections and improvements specific forms are required

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